

Heat transfer to a viscoelastic fluid in laminar flow through a rectangular channel

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Abstract—The measured local and mean Nusselt numbers for a viscoelastic fluid in laminar flow through a rectangular channel are found to be much higher than those of a purely viscous fluid or a Newtonian fluid. The differences cannot be explained on the basis of a superimposed free convection effect. Rather, the increase is due primarily to secondary flows which are induced in the viscoelastic fluid as a result of the normal force differences acting at the boundaries which are unique to elastic fluids. The pressure drop behavior is unaffected by the presence of secondary flows and predictions based on a purely viscous power law model give good agreement with the measured values.

INTRODUCTION

IT IS WELL known that the elastic properties of a viscoelastic fluid do not play a significant role in fully developed laminar pipe flow. In this case, the viscoelastic fluid behaves like a purely viscous non-Newtonian fluid and the friction factor and heat transfer coefficient may be predicted by the well known power law relations [1, 2]. For fully developed laminar pipe flow the friction factor is given by the following equation:

$$f = 16/Re' \quad (1)$$

where Re' is the generalized Reynolds number introduced by Metzner [1], defined as follows:

$$Re' = \rho V^{2-n} D^n \left[8^{n-1} K \left(\frac{3n+1}{4n} \right)^n \right]. \quad (2)$$

Here the values of n and K correspond to the power law relation

$$\tau_w = K(\dot{\gamma})^n. \quad (3)$$

In the case of heat transfer the fully established Nusselt number for the constant heat flux boundary condition is given by the following expression:

$$Nu_x = \frac{8(5n+1)(3n+1)}{31n^2 + 12n + 1}. \quad (4)$$

This expression is limited to values of x/D greater than $0.04 Pe$. In the entrance region the local Nusselt number for the constant heat flux case is given by:

$$Nu_x = 1.41 \left(\frac{3n+1}{4n} \right)^{1/3} Gz^{1/3}. \quad (5)$$

In contrast with the circular tube case there is evidence that the heat transfer behavior of a viscoelastic fluid in laminar flow through a non-circular geometry differs from that of a purely viscous fluid. Oliver [3] and

Oliver and Karim [4] experimentally found that viscoelastic fluids in laminar flow through flattened tubes gave higher heat transfer coefficients than Newtonian fluids of the same Prandtl number. Mena *et al.* [5] also reported higher heat transfer coefficients for a viscoelastic fluid in laminar flow through rectangular and triangular ducts as compared to a Newtonian fluid. However, they found no major effect of the viscoelasticity on the flow rate as compared to a Newtonian fluid. As a consequence, Mena concluded that the laminar flow of viscoelastic fluids through non-circular geometries is accompanied by secondary flows arising from elastic effects which have significant effects on the heat transfer, but not on the friction factor. Against this background, it was decided to carry out a set of careful pressure drop and heat transfer measurements in a rectangular duct with an aspect ratio of 0.5 using an aqueous polyacrylamide solution with well-defined shear-rate-dependent viscosity values.

EXPERIMENTAL SET-UP

The test section, shown in Fig. 1, is a 1.8×0.9 cm rectangular duct, 640 cm in length, with a hydraulic diameter of 1.2 cm. The top and bottom walls were made from 0.635-cm-thick stainless-steel plates while the narrow side walls were made from 1.27-cm-thick polycarbonate sheet with a low thermal conductivity of $0.23 \text{ W m}^{-1} \text{ K}^{-1}$. Some 85 thermocouples were positioned at 23 axial locations along the upper and lower heated walls while 16 pressure taps were located along the plastic side walls. The entrance to the test section is sharp and the thermal and hydrodynamic entrance lengths develop simultaneously.

The test section was positioned in the flow loop shown on Fig. 2 and insulated with styrofoam beads. The aqueous polymer solution is prepared in the reservoir tank (capacity of 400 gallons) from which it flows into the constant head tank which feeds the positive displacement Moyno pump. The fluid is

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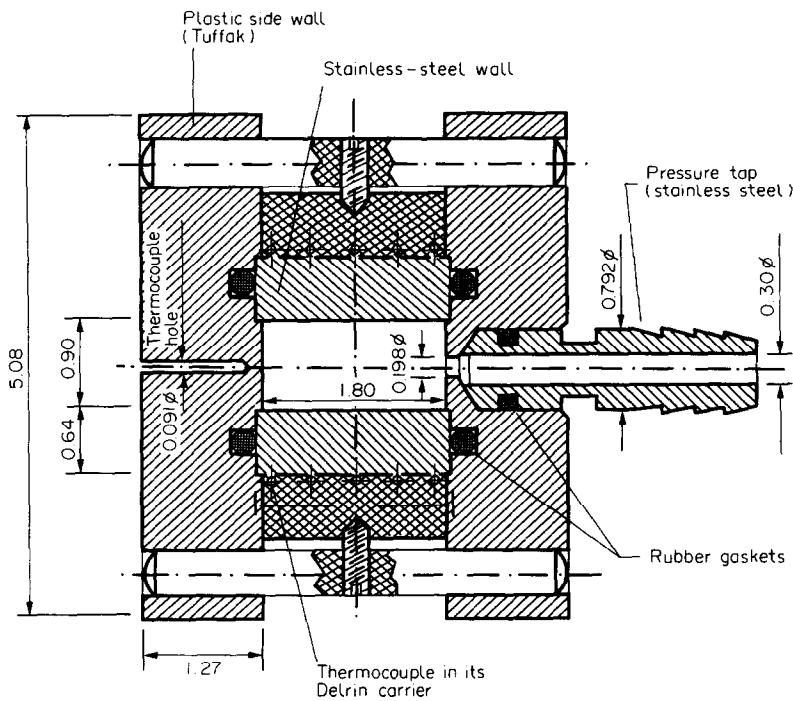


FIG. 1. Rectangular duct cross section with details of thermocouple and pressure tap attachment (dimensions are in cm).

- (4) The peripheral heat conduction is small compared to the radial conduction.
- (5) Heat losses to the surrounding are negligible.

The inner wall temperature is determined from the following relation:

$$T_{wi} = T_{wo} - (IE/WL\delta_w)\delta_w^2/2k_{st} \quad (6)$$

and the local bulk temperature is given by the linear equation

$$T_b = T_{b,in} + (T_{b,out} - T_{b,in})x/L. \quad (7)$$

The heat transfer per unit area is determined from the measured voltages and currents:

$$q_w = (IE)/(LW). \quad (8)$$

The local heat transfer coefficient follows directly

$$h_x = q_w/(T_{wi} - T_{b})_x. \quad (9)$$

RHEOLOGY OF THE VISCOELASTIC FLUID

It is well known that the flow and heat transfer characteristics of viscoelastic fluids are very sensitive to

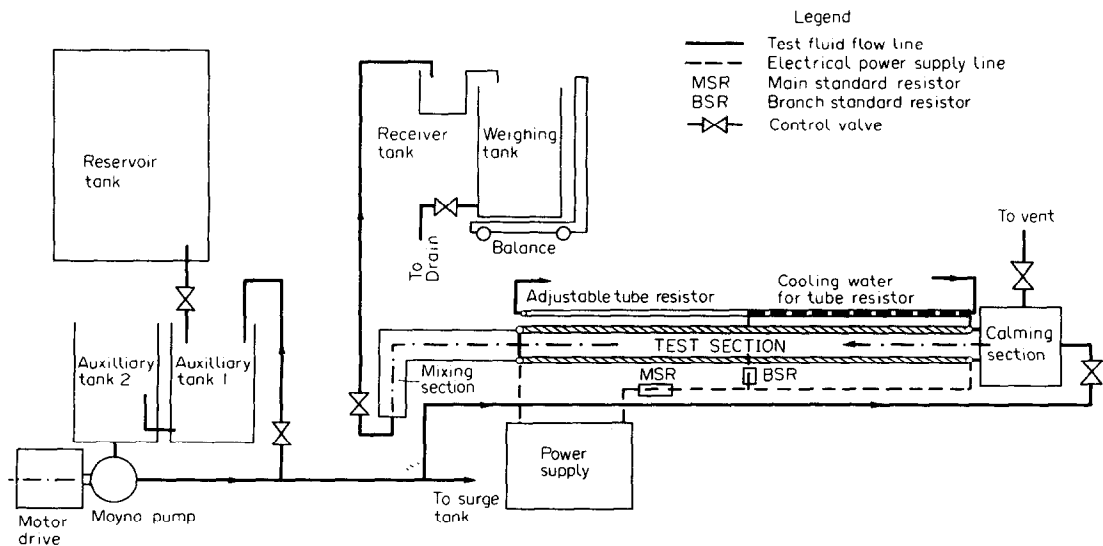


FIG. 2. Experimental set-up: flow and power supply loops.

the solute and solvent chemistry, the method of preparing the solution, aging and mechanical degradation. In light of these considerations special care was taken to use the same procedure for the preparation of each batch of test fluid. Furthermore, the viscosity of each test fluid was measured before and after the experiments in order to clearly define the test fluid. The viscoelastic fluid studied was a 1000 wppm polyacrylamide solution (Separan AP-273, DOW Chemical Co.) with the solvent being Chicago tap water having the chemical composition given in ref. [2]. The repeatability of the viscosity measurements of the viscoelastic fluids prepared for different runs was high. The measured shear stress τ_w (in N m^{-2}) as a function of the shear rate (in s^{-1}) may be fitted by the following polynomial

$$\log \tau_w = f(\log \dot{\gamma})$$
$$= a_0 + a_1 (\log \dot{\gamma}) + a_2 (\log \dot{\gamma})^2 + a_3 (\log \dot{\gamma})^3.$$

(10)

Typical values of the coefficients in equation 10 in the shear rate range from 0.03 to 30,000 s^{-1} are:

$$a_0 = -0.932; \quad a_1 = 0.621; \quad a_2 = -0.0688; \\ a_3 = 0.0195.$$

Now for any value of the shear rate it is easy to calculate the viscosity η , the power law exponent n and the consistency index K from the following relations:

$$\eta = \tau_w / \dot{\gamma} \tag{11}$$

$$n = d(\log \tau_w) / d(\log \dot{\gamma}) = a_1 + 2a_2 (\log \dot{\gamma}) + 3a_3 (\log \dot{\gamma})^2 \tag{12}$$

$$K = 10^{(\log \tau_w - n \log \dot{\gamma})} \tag{13}$$

Furthermore it is possible to determine the characteristic time of the fluid, λ , using the Powell–Eyring model (7)

$$\eta = \eta_\infty + (\eta_0 - \eta_\infty) \left(\frac{\sinh^{-1} \lambda \dot{\gamma}}{\lambda \dot{\gamma}} \right). \tag{14}$$

For the 1000 wppm aqueous polyacrylamide solutions used in the heat transfer runs the value of the characteristic time, λ , was of the order of 5 s. With this information, the dimensionless Weissenberg number, $\lambda V / D_h$, which is a measure of the elasticity of the fluid, can be determined for each test run.

FRICION FACTOR MEASUREMENTS

The friction factor measurements of the 1000 wppm aqueous polyacrylamide solution were carried out

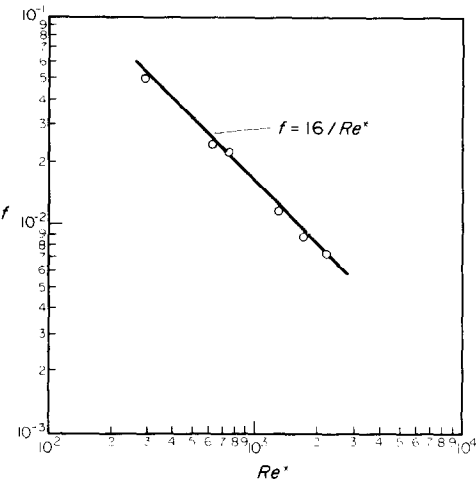


FIG. 3. Friction factors for fully developed laminar flow through a viscoelastic fluid in a 2:1 rectangular channel.

under adiabatic conditions at room temperature. The fully established friction factor measurements are shown on Fig. 3 and Table 2 as a function of the dimensionless Reynolds number, Re^* , introduced by Kozicki *et al.* [8]. Kozicki and his co-authors demonstrated that the fully established friction factor for laminar flow of a power law fluid through any constant cross-section channel can be given by the circular tube relation $f = 16 / Re^*$ where the Kozicki generalized Reynolds number, Re^* , is given by:

$$Re^* = \rho V^{2-n} D_h^n \left/ \left[8^{n-1} \left(\frac{a+bn}{n} \right)^n K \right] \right. \tag{15}$$

The constants a and b are dependent only on the geometry. It should be noted that Re^* reduces to Re' for laminar flow of a power law fluid through a circular tube. For a rectangular geometry the values are given in Table 1 as a function of the aspect ratio. For the 2:1 duct, the generalized Reynolds number becomes

$$Re^* = \rho V^{2-n} D_h^n \left/ \left[8^{n-1} \left(\frac{0.2440 + 0.7276n}{n} \right)^n K \right] \right. \tag{16}$$

Inspection of Fig. 3 and Table 2 indicates that the measured friction factor values are in reasonable agreement with the power law prediction, being some 5–6% lower than the prediction on the average.

The dimensionless pressure drop from the tube entrance to any position x along the rectangular

Table 1. Value of constants in equation (15)

Aspect ratio	0	0.25	0.50	0.075	1.0
<i>a</i>	0.5000	0.3212	0.2440	0.2178	0.2121
<i>b</i>	1.000	0.8182	0.7276	0.6866	0.6766

Table 2. Friction factor results for fully developed laminar flow of viscoelastic fluids without heating

Re^*	n	f_{exp}	$16/Re^*$	DEV (%)
297.0	0.542	0.04951	0.05395	-8.2
616.0	0.563	0.02414	0.02599	-7.1
749.0	0.577	0.02177	0.02135	2.0
1291.0	0.590	0.01161	0.01240	-6.3
1715.0	0.601	0.00875	0.00933	-6.2
2170.0	0.616	0.00735	0.00737	-0.3

Note: $f^* = 16/Re^*$, $DEV = 100(f_{exp} - f^*)/f^*$.

channel is given in Fig. 4. Sparrow and his colleagues carried out careful pressure drop measurements of several Newtonian fluids, including air, water and oil, in laminar flow through rectangular geometries [9, 10]. Their results may be expressed in terms of the generalized Reynolds number, Re^* , in the following form:

$$[p_{in} - p(x)]/(\rho V^2/2) = C(x) + 64(x/D_h)/Re^*. \quad (17)$$

Here $C(x)$, the dimensionless pressure drop in the entrance region, is a function of the aspect ratio and of the power law index n . The last term in equation (17), $64(x/D_h)/Re^*$, is consistent with the fully established friction factor prediction of $16/Re^*$. The current pressure drop measurements are in good agreement with the experimental values given by Beavers *et al.* [10] for air flow in a 2 : 1 duct. Since Curr *et al.* [11] and Collins and Schowalter [12] analytically demonstrate that $C(x)$ decreases with the power law exponent n , the experimental results are somewhat higher than expected for a purely viscous power law fluid with $n = 0.6$. However, there is some uncertainty in the value of $C(x)$. For example, the predicted limiting value of C for Newtonian flow in a 2 : 1 duct ranges from 1.1 to 1.8 (11, 13, 14, 15). Furthermore, the aqueous polymer solution is viscoelastic and there may be some secondary flow effects resulting from the normal force differences acting on the boundaries. Notwithstanding these uncertainties, it is apparent that the influence of elasticity on the pressure drop of a viscoelastic fluid is small and that, in general, the viscoelastic fluid behaves

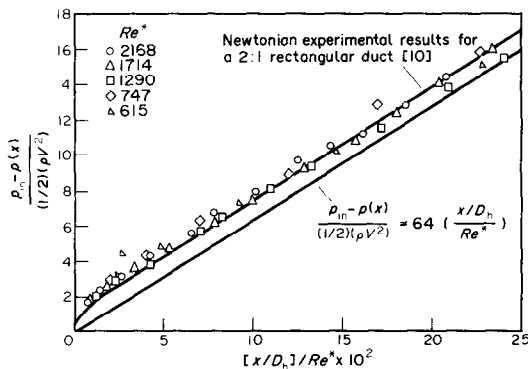


FIG. 4. Dimensionless pressure drop measurements for 1000 wppm aqueous polyacrylamide solutions in laminar flow through a 2 : 1 rectangular duct.

as a purely viscous non-Newtonian fluid. This is demonstrated by the reasonable agreement of the fully established friction factor with the power law prediction, $f = 16/Re^*$ and by the fair agreement of the measured dimensionless pressure drop with the predicted behavior of a power law fluid.

HEAT TRANSFER MEASUREMENTS

A series of heat transfer measurements was carried out with water as the test fluid prior to the main experiments with aqueous polyacrylamide solutions. The initial water runs involved heating of the upper wall only with the other three walls adiabatic. The local Nusselt values for this condition are given on Fig. 5(a). Subsequent water runs involved heating of the upper and lower surfaces with adiabatic side walls. In this case [Fig. 5(b)] there was clear evidence of natural convection being superposed on the forced convection and the lower wall Nusselt number reached values approximately twice those of the upper wall at values of x/D_h of the order of 200. Given the fact that the Rayleigh number, Ra , was of the order of 20,000–30,000 the large influence of natural convection is not surprising. The local Nusselt numbers are shown as a function of the Graetz number, mc_p/xk_f , in Fig. 6, which also presents some predictions of fully developed Nusselt values for the 2 : 1 rectangular geometry. The analytical results of Wibulswas for the simultaneous development of velocity and temperature of a Newtonian fluid with a Prandtl number of 10 in a 2 : 1 rectangular duct are also shown. (The thermal boundary condition is H_1 with all four walls heated.) It may be noted that the experimental Nusselt values for the upper plate are in good agreement with the analytical prediction of

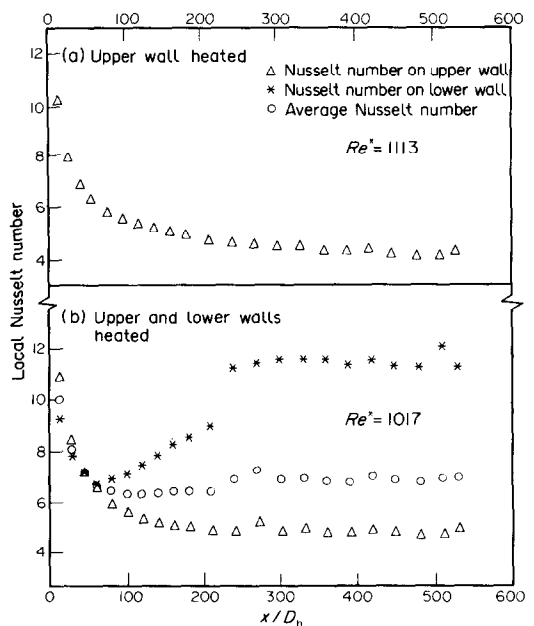


FIG. 5. Axial distribution of local Nusselt number of a typical run for laminar flow of water in 2 : 1 rectangular duct.

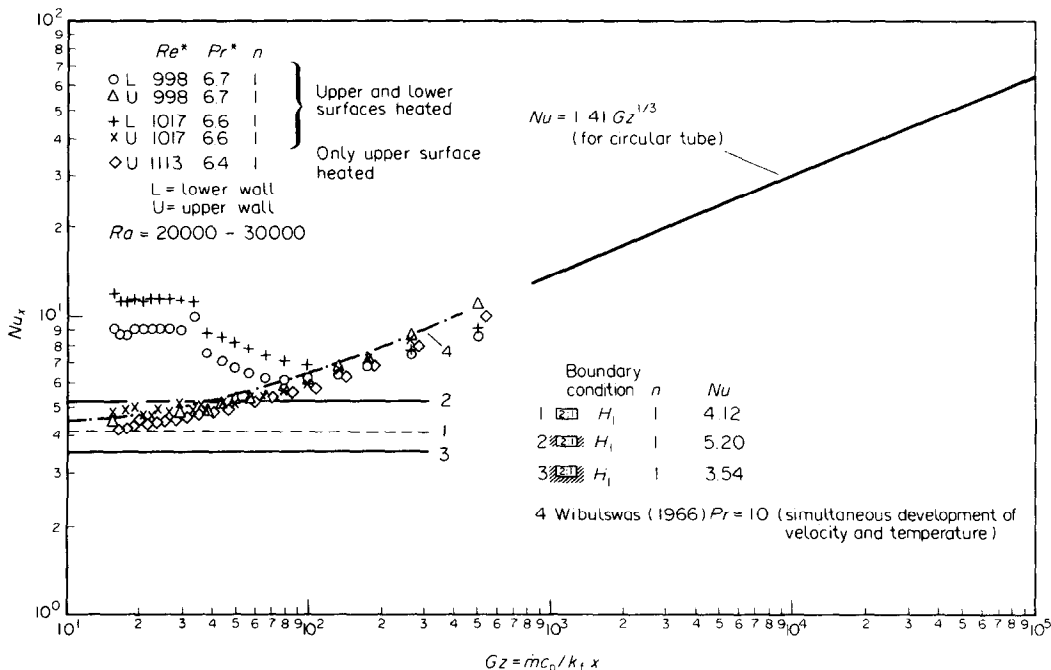


FIG. 6. Local Nusselt numbers for upper and lower surfaces vs Graetz number for laminar flow of water.

Wibelwas [16] which neglected any influence of free convection.

Turning next to the main series of heat transfer experiments with the viscoelastic aqueous polyacrylamide solutions (1000 wppm), Fig. 7 presents the measured axial temperature distributions along the outside of the upper and lower heated walls, the

adiabatic side walls, as well as the inlet and exit bulk fluid temperatures. Figure 8 shows a typical peripheral distribution of the outer wall and the side wall temperatures. For ease of presentation, the even-numbered stations are given on the right, while the odd-numbered stations are shown on the left. These measurements confirm the assumption that the upper and lower wall temperatures are constant at any axial position (although each wall is at a different temperature level), while the heat input per unit length is constant. The measurements also lend support to the assumption that the peripheral heat conduction is small compared to the radial conduction. The local Nusselt numbers along the upper and lower wall are shown for the representative run on Fig. 9, which also gives the local average or mean Nusselt number defined as $\bar{Nu}_x = 2/(1/Nu_{ux} + 1/Nu_{lx})$ (e.g. the mean Nusselt number uses the arithmetic average of the upper and lower wall-to-fluid temperature difference). A comparison of these results with those obtained with water shown on Fig. 5 reveals some major differences. First of all, the Nusselt numbers for the polyacrylamide solutions are much higher than those obtained with water. Furthermore, the behavior of the Nusselt number along the upper surface is much different in the two cases. In the water case, the Nusselt value for the upper wall is almost the same as for pure forced convection, while the lower plate shows evidence of mixed free and forced convection. In the case of the aqueous polyacrylamide solution, the Nusselt values along the upper wall are lower than those of the bottom wall, but the difference between these two values is much smaller than in the water runs. There is clear evidence that some type of secondary flow which enhances the heat transfer, occurs along both walls.

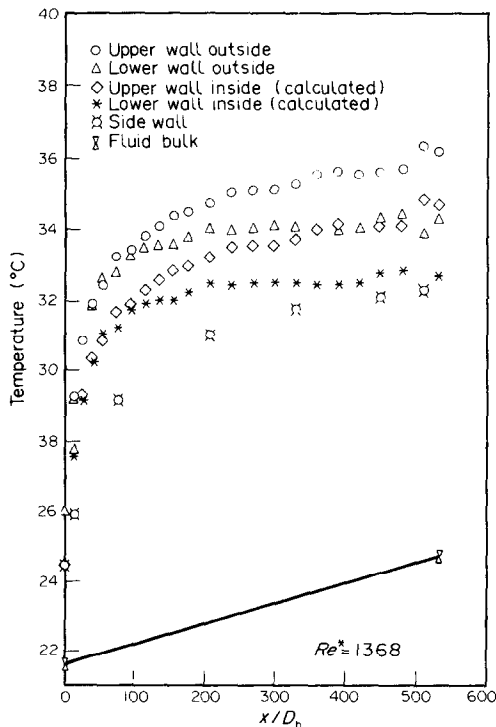


FIG. 7. Axial temperature distribution of a typical run for laminar viscoelastic fluid flow.

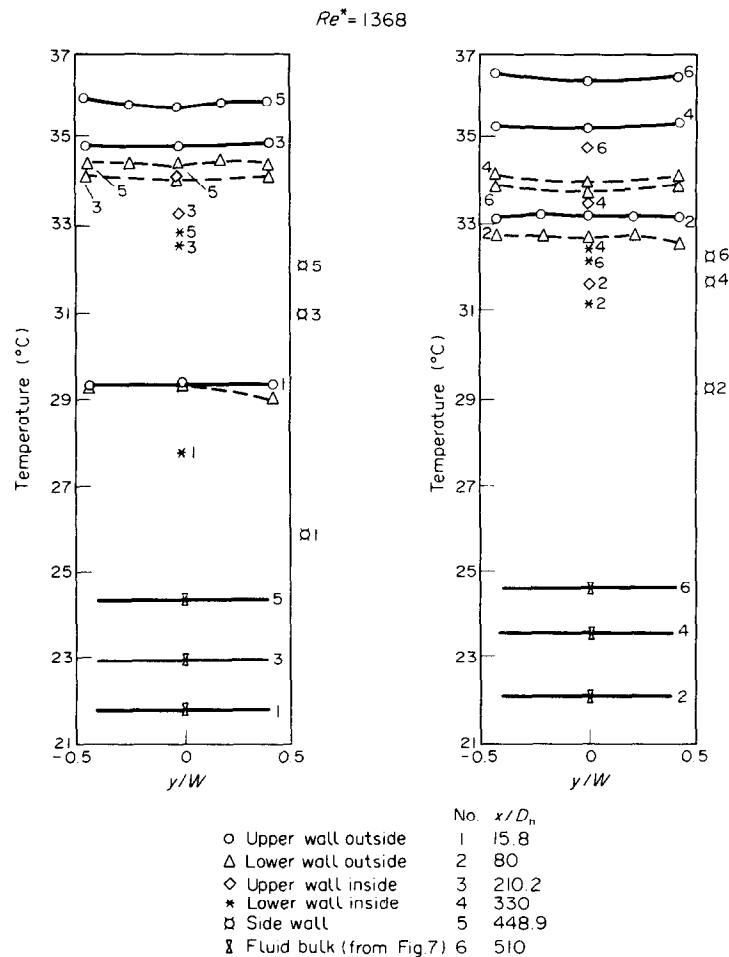


FIG. 8. Peripheral temperature distribution of a typical run for laminar viscoelastic fluid flow.

This is brought out more clearly in Fig. 10 which presents the experimental upper and lower wall Nusselt values as a function of Graetz number. Also shown are some predicted values of the Nusselt number for fully established thermal conditions. In the entrance region (e.g. at high Graetz number) the experimental values are in good agreement with forced convection predictions

for a power law fluid. It takes some distance from the beginning of heating before secondary effects, such as natural convection, begin to show up. Since the Rayleigh number of the heat transfer runs with the aqueous polyacrylamide solutions ranged from 5000 to 50,000 they bracketed the Rayleigh numbers for the water runs (20,000–30,000). A comparison of Fig. 10 with Fig. 6 suggests that the increased heat transfer found with the viscoelastic fluid cannot be accounted for solely on the basis of superimposed natural convection. Rather, it is hypothesized that the increased heat transfer is primarily due to secondary flows which arise from the normal force differences which occur on the boundaries of the viscoelastic fluid. (Such normal force differences do not occur in Newtonian fluids.) If this hypothesis is correct then the dimensionless heat transfer is a function of the Weissenberg number, a measure of the elastic effects, in addition to the usual dimensionless parameters which apply to Newtonian fluids. In the present aqueous polyacrylamide experiments the Weissenberg number based on the characteristic time evaluated by the Powell–Eyring equation ranged from 150 to 450. It is important to note that the friction factor behavior was

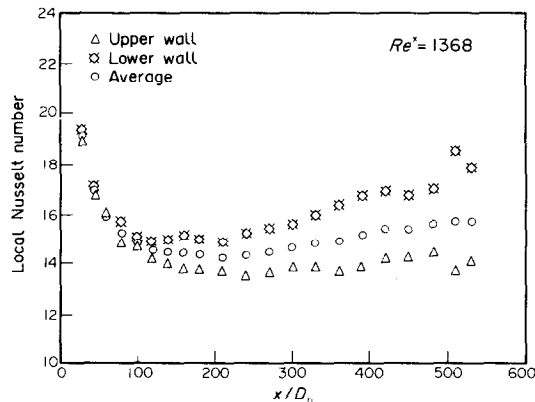


FIG. 9. Axial distribution of Nusselt number of a typical run for laminar viscoelastic fluid flow.

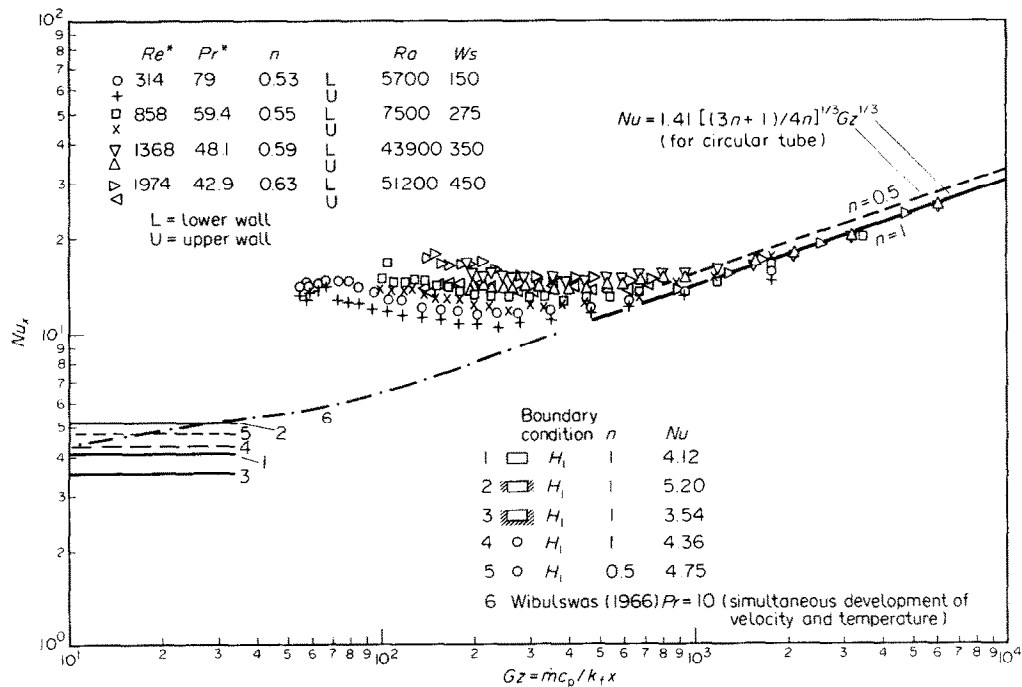


FIG. 10. Local Nusselt number vs Graetz number for laminar flow of viscoelastic fluid in a 2:1 rectangular duct.

relatively unaffected by the presence of secondary flows and the measured values were in good agreement with the values predicted for a purely viscous power law fluid.

The local mean Nusselt values, \overline{Nu}_x , for the water and

polyacrylamide solutions are summarized on Fig. 11, which also shows a number of predictions for the 2:1 rectangular geometry and the circular tube as well as the results of Wibuswas [16] for simultaneous development of the velocity and temperature of a

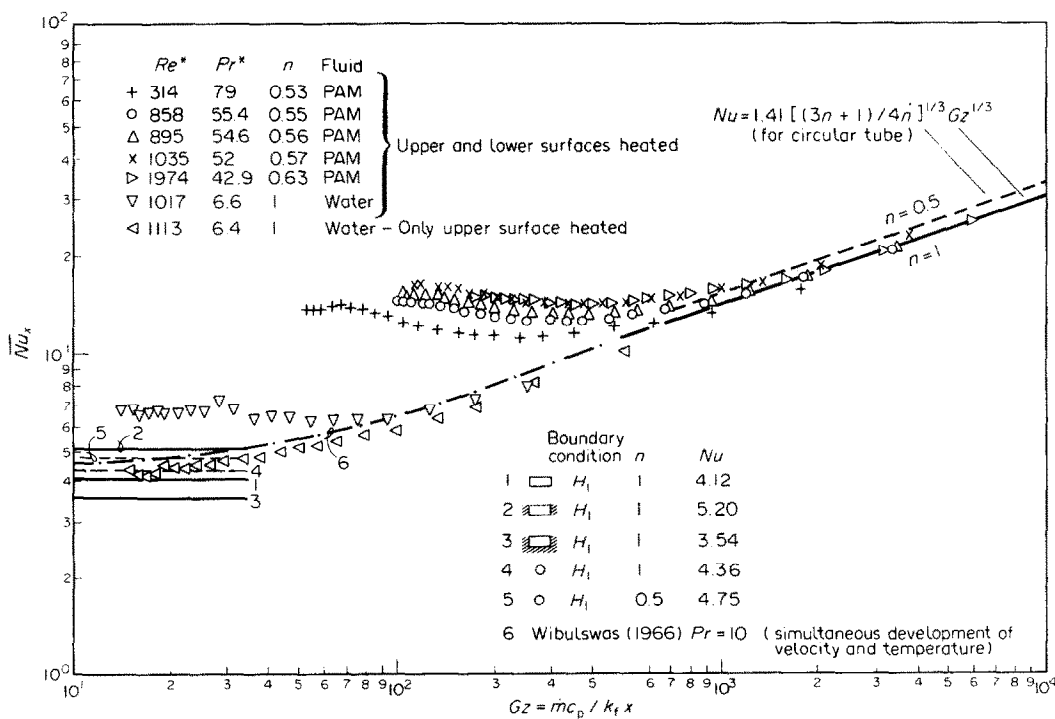


FIG. 11. Mean local Nusselt number vs Graetz number for laminar flow of viscoelastic fluid and of water in a 2:1 rectangular duct.

Newtonian fluid with a Prandtl number of 10 in a 2 : 1 rectangular duct. This figure brings out clearly that the dimensionless heat transfer is much higher for the viscoelastic fluid in laminar flow through a rectangular channel than in the corresponding case for a Newtonian fluid. Further, the influence of free convection cannot account for this difference. The influence of elasticity, giving rise to forces which generate secondary flows appears to be the most probable explanation of the increased heat transfer. These findings confirm and amplify the earlier conclusions of Oliver [3, 4] and Mena [5].

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TRANSFERT THERMIQUE POUR UN FLUIDE VISCOELASTIQUE EN ENCOULEMENT LAMINAIRE DANS UN CANAL RECTANGULAIRE

Résumé—Les nombres mesurés de Nusselt locaux et moyens pour un fluide viscoélastique en écoulement laminaire à travers un canal rectangulaire sont trouvés être plus élevés que ceux d'un fluide purement visqueux ou newtonien. Les différences ne peuvent pas être expliquées sur la base d'un effet de superposition de convection naturelle. L'accroissement est plutôt dû principalement à des écoulements secondaires induits dans le fluide viscoélastique par des forces normales aux parois, spécifiques des fluides élastiques. La chute de pression n'est pas affectée par la présence des écoulements secondaires et les prévisions basées sur un modèle à la loi puissance s'accordent bien avec les valeurs mesurées.

WÄRMEÜBERGANG AN EIN VISKOELASTISCHES FLUID BEI LAMINARER STRÖMUNG DURCH EINEN RECHTECKIGEN KANAL

Zusammenfassung—Die gemessenen örtlichen und mittleren Nusselt-Zahlen für ein viskoelastisches Fluid bei laminarer Strömung durch einen rechteckigen Kanal waren erheblich größer als Nusselt-Zahlen eines rein viskosen Fluides oder eines Newton'schen Fluides. Die Unterschiede können nicht durch einen überlagerten Effekt der freien Konvektion erklärt werden. Stattdessen resultiert die Verbesserung vorwiegend aus Sekundärströmungen, die in dem viskoelastischen Fluid durch Normalkräfte an den Begrenzungen hervorgerufen werden. Diese sind für elastische Fluide gleichförmig verteilt. Der Druckabfall wird durch die Gegenwart von Sekundärströmungen nicht beeinflusst, und Berechnungen aufgrund eines Potenzansatzes für rein viskose Fluide liefern eine gute Übereinstimmung mit den gemessenen Werten.

ТЕПЛОПЕРЕНОС К ВЯЗКОУПРУГОЙ ЖИДКОСТИ ПРИ ЛАМИНАРНОМ ТЕЧЕНИИ ЧЕРЕЗ КАНАЛ ПРЯМОУГОЛЬНОЙ ФОРМЫ

Аннотация—Найдено, что измеренные локальное и среднее числа Нуссельта для вязкоупругой жидкости при ламинарном течении в прямоугольном канале значительно выше, чем для чисто вязкой или ньютоновской жидкости. Это различие не могло быть объяснено с позиций влияния свободноконвективного теплообмена. Скорее рост вызван прежде всего вторичными течениями, которые возникают в вязкоупругой жидкости в результате действия сил, нормальных границам, которые характерны для упругих жидкостей. Наличие вторичных течений не влияет на перепад давления, а результаты, основанные на степенной модели чисто вязкой жидкости, хорошо согласуются с данными измерений.